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Analysis of fuzzy thresholding schemes

C.V. Jawahar*, P.K. Biswas, A.K. Ray

Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur, West Bengal, India - 721 302

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Abstract

Fuzzy thresholding schemes preserve the structural details embedded in the original gray distribution. In this paper, various fuzzy thresholding schemes are analysed in detail. Thresholding scheme based on fuzzy clustering has been extended to a possibilistic framework. The characteristic difference for assignment of membership of fuzzy algorithms and their correspondence with conventional hard thresholding schemes have been investigated. A possible direction towards unifying a number of hard and fuzzy thresholding schemes has been presented. © 2000 Pattern Recognition Society. Published by Elsevier Science Ltd. All rights reserved.

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1. Introduction

Digital image segmentation, one of the most challenging problems in image processing, is very frequently attempted with pattern recognition methods. Segmentation is the process of partitioning an image into a finite set of regions, such that a distinct and well-defined property is associated with each of them. Thresholding, the simplest and most popular strategy for segmentation, refers to the process of partitioning the pixels in an image $\mathscr{I} = [I_{mn}]_{M \times N}$, $I_{mn} \in \mathbf{L} = \{1, 2, \ldots, L\}$, defined over a two dimensional grid $G = (m, n), 0 \leqslant m \leqslant M - 1, 0 \leqslant n \leqslant N - 1$ into object (O) and background (B) regions i.e.,

$$O = \{(m, n) | I_{mn} \geqslant T\}$$
$$B = \{(m, n) | I_{mn} < T\}, \tag{1}$$

in such a way that $O \cup B = G$ and $O \cap B = \emptyset$. Here, T, the discriminant gray value is the hard threshold.

Identification of an optimal threshold T is a complex task. A number of elegant algorithms are proposed for this purpose. They are based on region separability, minimum error, entropy, etc. [1–3]. Another important class of algorithms employ scale-space theory [4,5] for thresholding. Most of these algorithms are initially meant for binary thresholding. This binary thresholding procedure may be extended to a multi-level one with the help of multiple thresholds $T_1, T_2, \ldots, T_n$ to segment the image into $n + 1$ regions [6,7]. Multi-level thresholding based on a multi-dimensional histogram resembles the image segmentation algorithms based on pattern clustering.

A hard dichotomization of pixels as in Eq. (1) is extremely difficult when boundaries are fuzzy and regions are ill defined, which is frequently the case in image analysis. Moreover, the imprecision of gray values and vagueness in various image definitions make the segmentation problem more difficult to manage with deterministic or stochastic image processing schemes. This led to the development of a number of algorithms based on fuzzy set-theoretic concepts [8–10].

Fuzzy thresholding involves the partitioning of an image into two fuzzy sets, i.e., $\tilde{O}$ and $\tilde{B}$, corresponding to object and background regions by identifying the membership distributions $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$ associated with them. A natural extension of Eq. (1) into fuzzy setting was carried out by Pal [10,11] by defining a "bright image" characterised by a monotonic membership function $\mu_{\tilde{O}}$,

* Corresponding author. Centre for Artificial Intelligence and Robotics, Raj Bhavan Circle, High Grounds, Bangalore - 560 001, India.

E-mail address: jawahar@cair.res.in (C.V. Jawahar).

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such that

$$\mu_{\tilde{O}}(I_{mn}) < 0.5 \quad \text{if } I_{mn} < T,$$

$$\mu_{\tilde{O}}(I_{mn}) > 0.5 \quad \text{if } I_{mn} > T, \tag{2}$$

and the crossover point of the membership function corresponds with the hard threshold T. Background region, $\tilde{B}$, was considered as the complement of object region, $\tilde{O}$, i.e.,

$$\mu_{\tilde{B}}(j) + \mu_{\tilde{O}}(j) = 1.0 \quad \forall_j \in \mathbf{L}. \tag{3}$$

They preferred to assign the membership with a standard S function [11,12].

Huang and Wang [13] proposed a fuzzy thresholding scheme which minimises the fuzziness in the thresholded description and, at the same time, accommodates the variations in the gray values within each of the regions. They assigned memberships as

$$\mu_{\tilde{O}}(I_{mn}) = \frac{1}{1 + |v_{\tilde{O}} - I_{mn}|/C} \quad \text{if } I_{mn} \geqslant T$$

and

$$\mu_{\tilde{O}}(I_{mn}) = 0 \quad \text{if } I_{mn} < T, \tag{4}$$

where $v_{\tilde{O}}$ is the mean gray value of the fuzzy object region, $\tilde{O}$, and the parameter C controls the amount of fuzziness in the thresholded description. A similar membership assignment is employed for $\tilde{B}$ also. They classified the pixels unequivocally into object or background regions with the help of a hard threshold T and thereby led to an abrupt discontinuity of membership distribution in object and background regions, such that

$$\tilde{O} \cup \tilde{B} \subseteq G \quad \text{and} \quad \tilde{O} \cap \tilde{B} \supseteq \emptyset. \tag{5}$$

In our earlier paper [14], investigations are reported on the suitability of fuzzy clustering formulation for the fuzzy thresholding process. The appropriateness of fuzzy clustering schemes for thresholding can be asserted with the fact that an optimal fuzzy partition based on fuzzy clustering depicts the substructure embedded in the data set and reflects the gray distribution within the object and background regions.

All these formulations assume that the difference in gray level alone leads to two visually apparent distinct regions, and the gray-level histograms are characterised with two modes which may be closer or far and/or may have different sizes. These geometrical and statistical characteristics of the histogram play an important role in threshold identification. In this case, histogram is often expected to be of the form

$$h_j = f(d(j, v_1), \sigma_1) + \rho f(d(j, v_2), \sigma_2), \quad j \in \mathbf{L}, \tag{6}$$

where $\mathrm{H} = \{h_j\}$ is the histogram of $\mathscr{I}$ with h_j denoting the frequency of occurrence of gray value j. Here ρ corresponds to the ratio of the sizes of object and background regions in the image while another parameter $\gamma = \sigma_2/\sigma_1$ denotes the ratio of scatters and the suffixes 1 and 2 represent the regions $\tilde{B}$ and $\tilde{O}$, respectively. It is assumed here that the "true" object and background gray values are perturbed by a physical process to form a continuous non-negative function $f(\cdot)$ of gray values with continuous derivatives.

The objective of this paper is to analyse the fuzzy thresholding process based on the membership assignment characteristics, and their correspondence with the classical hard thresholding schemes. A fuzzy thresholding procedure based on possibilistic clustering is proposed, and the implementation aspects of fuzzy thresholding algorithms are discussed. Analysis is carried out on the capabilities of various algorithms to reflect the structural details of the gray distribution of the original image. A step towards unifying various thresholding schemes are also presented.

2. Thresholding based on soft partitioning

Since thresholding is basically a pixel classification problem, fuzzy thresholding formulations are found to be appropriate for this task. In this section, we briefly explain the thresholding procedure based on fuzzy clustering [15] and extend it with the help of possibilistic concepts.

The problem of fuzzy clustering is that of partitioning a set of n points $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ into c classes, i.e., $\omega_1, \omega_2, \ldots, \omega_c$, such that

$$\text{(a) } \mu_i(x_j) \in [0, 1] \quad \text{(b) } 0 < \sum_{j=1}^{n} \mu_i(x_j) < n \quad \text{and}$$

$$\text{(c) } \sum_{i=1}^{c} \mu_i(x_j) = 1.0, \tag{7}$$

where $\mu_i(x_j)$ is the membership of x_j in the ith class ω_i.

A natural extension of fuzzy clustering for segmentation by considering the gray value alone as a feature leads to the thresholding formulation. For thresholding, the fuzzy c-means-based objective function to be minimised is

$$J(H, \mu_{\tilde{B}}, \mu_{\tilde{O}}) = \sum_{i=1}^{2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mu_i(I_{mn})^{\tau} d^2(v_i, I_{mn})$$

$$= \sum_{i=1}^{2} \sum_{j=1}^{L} h_j \mu_i(j)^{\tau} d(v_i, j)^2. \tag{8}$$

Thresholding algorithm assumes an initial partition and goes on iteratively evaluating the region means as

$$v_i = \frac{\sum_{j=1}^{L} j h_j \mu_i^{\tau}(j)}{\sum_{j=1}^{L} h_j \mu_i^{\tau}(j)} \tag{9}$$

and the memberships using

$$\mu_{\tilde{O}}(j) = \frac{1}{1 + (d(v_{\tilde{O}}, j)/d(v_{\tilde{B}}, j))^{2/(\tau - 1)}}$$

$$\text{and} \quad \mu_{\tilde{B}}(j) = 1 - \mu_{\tilde{O}}(j) \tag{10}$$

until there is no appreciable change in the partition. More details of this fuzzy thresholding scheme based on fuzzy c-means (fcm) and its variants to segment images having imbalance in size and scatter of object and background regions are discussed in our earlier paper [14].

Possibilistic clustering algorithm, proposed by Krishnapuram and Keller [16], based on the possibilistic theory, relaxes constraint (7c) of fuzzy partition and provides a soft description of clusters where $\mu_i(x_j) \in [0, 1]$ denotes the compatibility of element x_j to the ith region.

Here, thresholding based on possibilistic c-means algorithm minimises an objective function

$$J(H, \mu_{\tilde{O}}, \mu_{\tilde{B}}) = \sum_{i=1}^{2} \sum_{j=1}^{L} h_j \mu_i(j)^\tau d(v_i, j)^2 + \sum_{i=1}^{2} \eta i \sum_{j=1}^{L} (1 - \mu i(j))^\tau. \tag{11}$$

which is similar to Eq. (8) with an additional term to avoid the trivial solution due to the relaxation of constraint (7c). The parameter $\eta_i \in \mathbb{R}^+$ may be the same for all i or it may be estimated for each of the clusters. The modified objective function along with the constraints i.e Eqs. 7(a) and 7(b) provide the membership updation formulae, which assumes the following form for binary thresholding, i.e, when $c = 2$.

$$\mu_{\tilde{O}}(j) = \frac{1}{1 + (d(v_{\tilde{O}}, j)^2/\eta_1)^{1/(r-1)}}; \quad \text{and}$$

$$\mu_{\tilde{B}}(j) = \frac{1}{1 + (d(v_{\tilde{B}}, j)^2/\eta_2)^{1/(r-1)}}. \tag{12}$$

Similar to the thresholding scheme based on fcm, fuzzy thresholding algorithm based on possibilistic clustering assumes an initial partition and iteratively evaluates the memberships until there is no appreciable change in the partition. In short, these fuzzy thresholding schemes yield a soft partition while minimising the criterion function.

3. Iterative and non-iterative implementation

There are mainly two approaches to implement the criteria-based thresholding schemes—either by searching the extrema for all possible thresholds or by identifying the optimal combination by iteratively evaluating the criteria function and updating the threshold accordingly. As may be seen from the formulations based on Eq. (8) or Eq. (11), the problem of fuzzy thresholding is that of identification of the minima of $J(\cdot)$ to obtain the optimal thresholded description from fuzzy and possibilistic sense. Since the memberships are assigned according to Eqs. (10) and (12) in these two cases, the problem of fuzzy thresholding reduces to the identification of the minima of $J(\cdot)$ for various combinations of v_1 and v_2. Unlike in the hard thresholding cases, where a search for an extrema of a criterion function is carried out by varying the threshold T alone, fuzzy thresholding requires the variation of v_1 and v_2 to obtain the optimal partition.

In general, the non-iterative implementation of a fuzzy thresholding algorithm requires the following steps.

for all possible ξ_1
    for all possible ξ_2
    {
        Assign memberships $\mu_{\tilde{B}}$ *and* $\mu_{\tilde{O}}$
        Compute the criteria function $J(\cdot)$
    }
Identify the membership distributions corresponding to the extrema of $J(\cdot)$.

Here ξ_1 and ξ_2 correspond to parameter vectors associated with the background and object regions. They are v_1 and v_2 in case of the fuzzy thresholding schemes discussed in the previous section. An alternate iterative formulation closely follows the following steps:

1. Initialise the thresholded description $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$ satisfying Eq. (3).
2. Compute the mean gray-values of both the regions using Eq. (9).
3. Assign the membership values using Eq. (10) (or Eq. (12)).
4. Repeat steps 2–4 until there is no appreciable change for $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$.

A look on the prospects of the iterative and the non-iterative implementation of the proposed algorithms supports the efficient iterative implementation, provided it converges. Since the convergence of fuzzy c-means and possibilistic c-means are proved in literature [15,16], the following lemma may be stated:

Lemma. *Fuzzy thresholding based on soft clustering algorithms converge to the minima of the objective function Eq. (8) (or Eq. (11)) with repeated updation of memberships with Eq. (10) (or Eq. (12)) and the evaluation of regional mean gray-values using Eq. (9).*

Since the formulae for updation of memberships and computation of regional mean gray values are derived out of fuzzy and possibilistic clustering formulation, the algorithm is only a simple Picard iteration where each step minimises the objective function with respect to only one parameter by keeping the other independent parameters fixed and thus lead to a minima of the objective function.

4. Hardening of fuzzy thresholded description

Often the interest of image analysis methodologies restricts to the extraction of the object from a scene so as to characterise the object with a set of features. Even though a fuzzy thresholded description is sufficient for this purpose, conventional feature extraction and object recognition methods may not be applicable as such with this description. Thus in spite of the presence of elegant image analysis techniques developed based on fuzzy thresholded (or segmented) description, hardening schemes are required to make the description useful for the conventional object recognition schemes. Typical hardening schemes are proposed below.

Scheme: HARD-1: A simple method may be hardening the fuzzy thresholded description using Eq. (1) with a crisp threshold T_f, such that $v_1 < T_f < v_2$ and $\mu_{\tilde{O}}(T_f) = \mu_{\tilde{B}}(T_f)$. This provides mutually exclusive and exhaustive object and background regions.

Scheme: HARD-2: Another possible hardening procedure is based on α-cuts of fuzzy sets as

$$O = \tilde{O}_\alpha = \{(m,n);\ \mu_{\tilde{O}}(I_{mn}) \geqslant \alpha\} \quad \text{and} \quad B = G - O. \tag{13}$$

The parameter $\alpha \in (0,1]$ directly controls the size of the object region. As α increases, O approaches the core/skeleton of the object region.

Scheme: HARD-3: Both the above hardening schemes are well applicable for thresholding, yet, they find difficulty to extend to a general fuzzy segmentation scheme, where the number of classes is more than two and feature space is multidimensional with non-linear decision boundary. In such a case, one may harden the classes as

$$\omega_i = \{x;\ \mu_{\tilde{\omega}i}(x) > \mu_{\tilde{\omega}j}(x)\ \forall j \neq i\}. \tag{14}$$

Indeed, there may be cases with $\mu_{\tilde{\omega}i}(x) = \mu_{\tilde{\omega}j}(x)$ where x may be assigned arbitrarily to ω_i or ω_j according to an appropriate heuristic. Note that, such cases are of more academic interest than of practical significance.

Scheme: HARD-4: In cases where a deterministic misclassification is very costly hardening may be carried out as

$$\omega_i = \{x;\ \mu_{\tilde{\omega}i}(x) > \mu_{\tilde{\omega}j}(x) \quad \text{and} \quad \mu_{\tilde{\omega}i}(x) \geqslant \beta\ \forall j \neq i\}. \tag{15}$$

In this case, some of the points may remain as unlabelled, according to the parameter $\beta \in [0,1]$, even after hardening.

These hardening schemes may be employed to validate the applicability of fuzzy thresholding schemes by comparing with the hard thresholded description. An image, shown in Fig. 1a, is thresholded using the fuzzy thresholding scheme-based ob fuzzy c-means. The resulting fuzzy thresholded description, is hardened with scheme HARD-2 Eq. (13). The hardened description for $\alpha = 0.9$, 0.5 and 0.1 are shown in Fig. 1(b), (c), (d), respectively. From a single-thresholded description, the variability in membership distribution provides a set of hard representations with the variation of compatibility of pixels to the object region. It may be noted that a good amount of structural information is present in the fuzzy thresholded description, which is not available in the classical hard descriptions. This leads to the following proposition.

Proposition. *A set of hard thresholded descriptions is embedded in a fuzzy thresholded description characterised by a membership distribution* $\mu\tilde{B}$ *and* $\mu_{\tilde{O}} \in [0,1]$ *since* $\tilde{O} = \cup_\alpha \alpha \tilde{O}_\alpha$.

Therefore, the fuzzy thresholding schemes provide the advantage of better and detailed representation of the intra-region gray distribution. Thus the fuzzy thresholding formulations provide very useful information for the high-level vision

5. Membership assignment philosophy

As fuzzy sets are represented by membership functions, fuzzy thresholding schemes may be characterised by the membership assignment philosophy. The performances of all the reported thresholding algorithms depend, to a large extent, on the underlying assumptions behind their formulations. Thresholding is one of the most preferable segmentation method, if two distinct regions are apparently existing and the perturbations around the mean gray value of the object and background regions provide a gray-level picture with bimodal histogram. In this case, the membership assignment scheme should assign maximum membership grades to the mean values of the object and the background regions and the membership should decrease monotonically as the gray level distance from the respective means increases. This kind of a membership function reflects the true nature of object and background geometries, referred to as the structural details of the regions in the context of pattern recognition. Most of the reported global thresholding schemes based on histogram perform extremely well if the object and background regions are generated by identical gray distributions, i.e., both object and background gray distributions are equal in size as well as in scatter. Histogram is considered as the addition of these two distributions. A comparison of membership distribution will throw some light on the qualitative characteristics of various methods. Here two existing fuzzy thresholding schemes with distinct philosophies and the two schemes based on fuzzy and possibilistic clustering algorithm are compared. To compare the membership assignment philosophy of all the four algorithms, a bimodal histogram

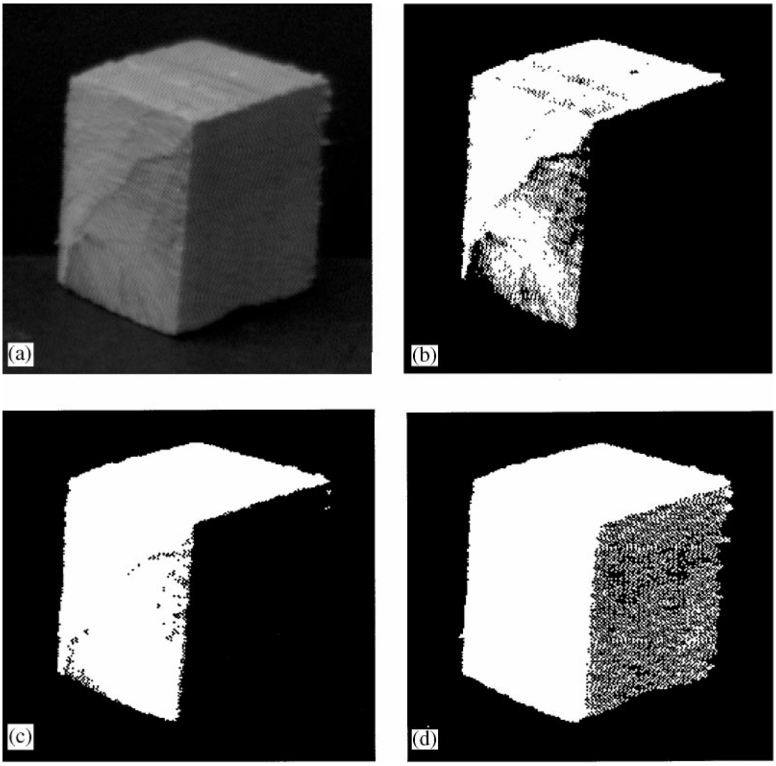

Fig. 1. (a) Original image and its fuzzy thresholded description hardened with $\alpha =$ (b) 0.9, (c) 0.5 and (d) 0.1.

has been considered with identical well separated modes as shown in Fig. 2. The valley of the histogram T_v is the optimal hard threshold.

The membership distribution of object and background regions have been computed using four algorithms viz., [10] (Fig. 3a), [13] (Fig. 3b), fuzzy c-means (Fig. 3c) and possibilistic c-means (Fig. 3d). It may be seen here that, the algorithm by Murthy and Pal identifies the fuzziness in the transition region quite effectively. The other algorithms reflect the structural detail embedded in the scene in a better fashion. Even though Huang and Wang considered the regions as fuzzy, the region of support of object and background regions are found to be mutually exclusive. At the same time, the segmented descriptions based on fuzzy and possibilistic clustering provide fuzzy descriptions with continuous variation of memberships.

The difference between the membership assignment schemes of fuzzy and possibilistic clustering is due to the orthogonality constraint Eq. (7) which is relaxed in the latter case. While the fuzzy partition preserves the relative geometrical structure, the possibilistic partition, on the other hand, provides the absolute geometrical details of the scene. It may be noted that the algorithm of Murthy and Pal as well as the fuzzy c-means algorithm provide description where Eq. (3) is satisfied. While the

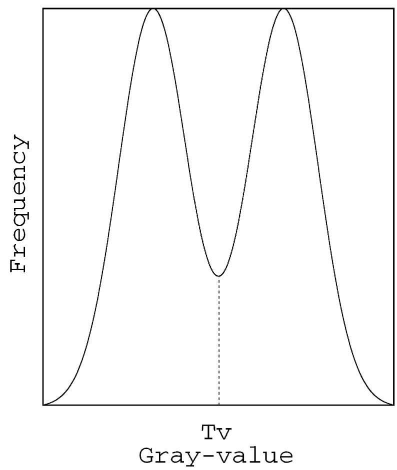

Fig. 2. A histogram with equal size and scatter of background and object regions.

method proposed by Huang and Wang and possibilistic clustering satisfy

$$\mu_{\tilde{B}}(j) + \mu_{\tilde{O}}(j) \in (0,1] \quad \text{and} \quad \mu_{\tilde{B}}(j) + \mu_{\tilde{O}}(j) \in (0,2],$$

respectively.

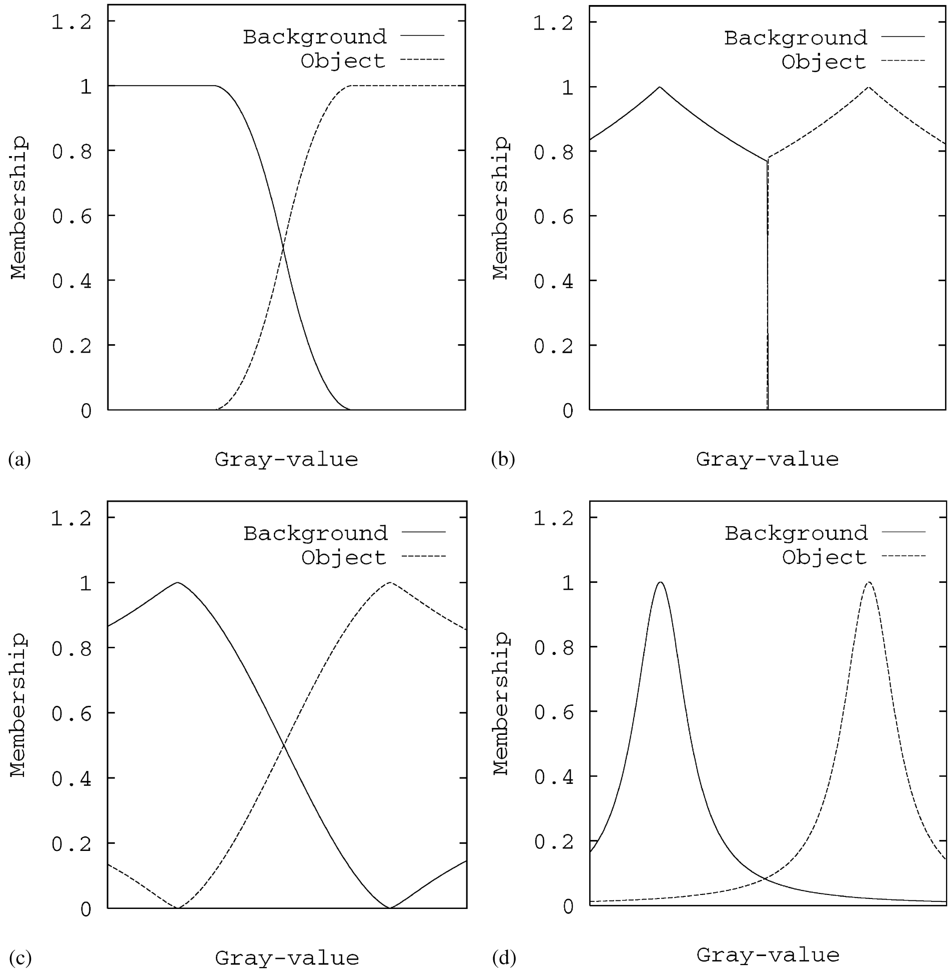

Fig. 3. Membership assignments using (a) Murthy and Pal (b) Huang and Wang (c) Fuzzy c-means (d) Possibilistic c-means.

6. Analysis of formulations

Another important aspect from which fuzzy thresholding algorithms has to be seen is the basis of formulation. Though they differ widely in their implementation and results, interestingly, their philosophical origin coincides. It is well known from the cluster validity literature [15] that a particular fuzzy partition is preferred if it is the hardest of all the possible fuzzy partitions with the same set of parameters. The same idea is extended for fuzzy thresholding by minimisation of gray-level fuzziness in Huang and wang [13] as well as in Murthy and Pal [10]. They have employed various measures of fuzziness, such as index of fuzziness, fuzzy entropy, etc., and have identified the optimal threshold as the minima of these measures with the help of an extensive search. Fuzzy entropy, the most popular measure of fuzziness is a scalar measure of a fuzzy set as given below:

$$-\mu(j)\log(\mu(j)) - (1-\mu(j))\log(1-\mu(j)). \quad (16)$$

Analytical comparison of the thresholding schemes formulated from diverse points of view, to perform the same task, is extremely difficult. From this aspect, it will be worth to observing that, there are common philosophical basis for all these algorithms. Recently, such an attempt has received attention. Many of the cost functions employed by different algorithms may be considered as closely related. A study of such a unification, limited to three popular thresholding schemes proposed in literature [1,3,13], has been reported in [17]. Another possible common measure may be an information-theoretic measure.

It is quite apparent from the formulation itself that thresholding algorithms search for a structure of the gray distributions within the object and background regions. Entropy provides a measure of deviation of a distribution from a well-defined structure and is useful for such a search [18]. The definition of entropy may vary from case to case. In a probabilistic environment, entropy becomes maximum if $p_1 = 1/L \; \forall i$, while in a fuzzy

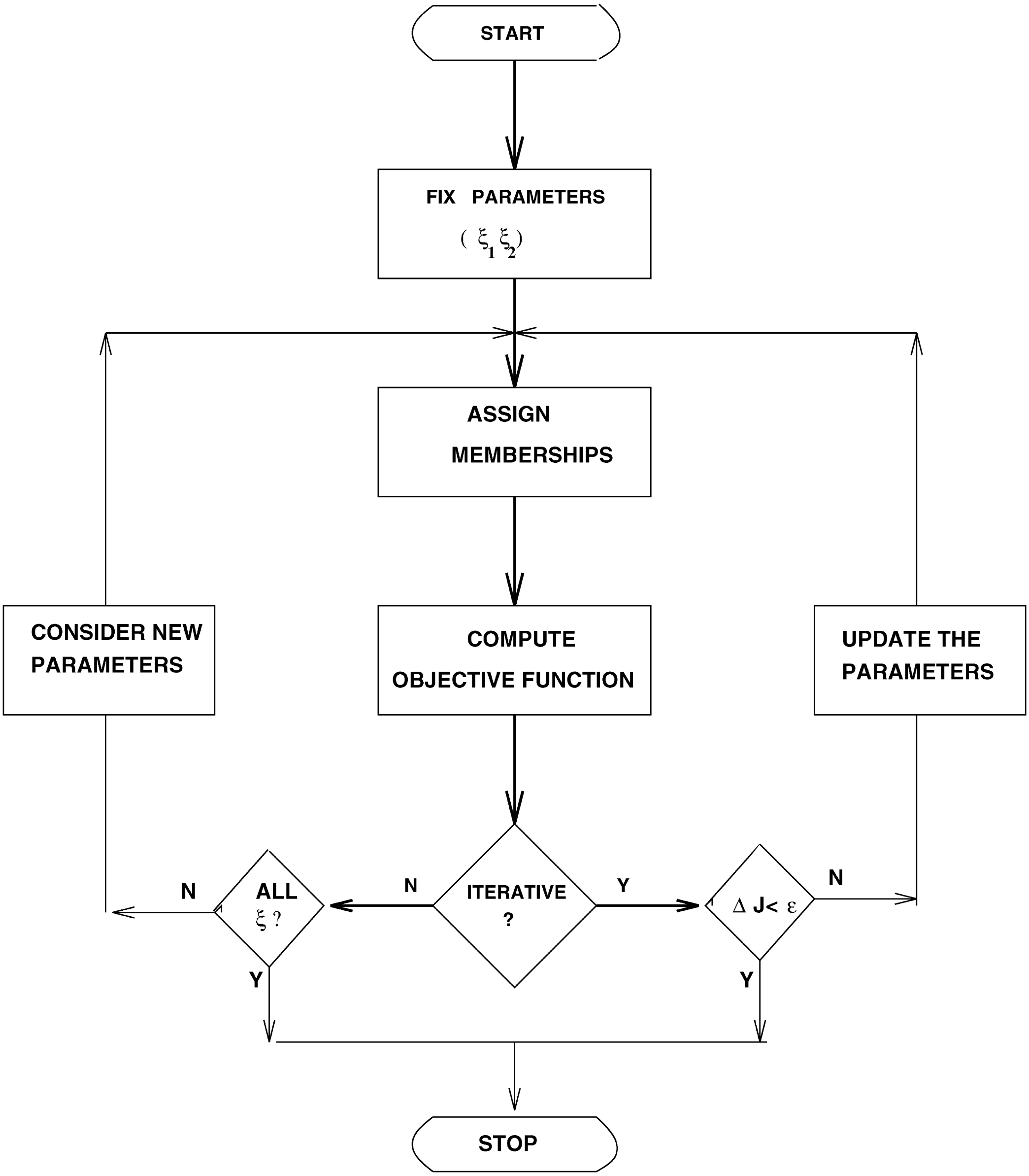

Fig. 4. Fuzzy thresholding procedure.

environment, entropy [19] reflects the uncertainty in the distribution and is maximum if the set and its complement equals i.e., $\mu_{\tilde{O}}(j) = \mu_{\tilde{B}}(j)$. In fact, it is not essential to depend on these logarithmic definitions of entropy for the present purpose. An algebraic definition of entropy [18,20], such as the deviation of the given distribution from a standard one such as $f((p_i - q_i)^2)$, is also applicable.

Based on the above discussion, it may be possible to unify a number of criteria function-based thresholding schemes into a single one.

6.1. Generalisation of formulations

In the reported algorithms [14] 1 and 2, it is the distortion (perturbation) of the gray values from the mean of the regional gray distributions, which is minimised, while in algorithm 3, [14], the deviation from the modelled gray distribution with Gaussian function is minimised. In all the cases, the memberships are assigned such that, the thresholded description is less fuzzy or more hard; in other words, the fuzzy entropy is relatively less.

In general, irrespective of whether iterative or non-iterative, a typical fuzzy thresholding scheme assume the steps shown in Fig. 4. Basically, these algorithms consider the optimal fuzzy thresholded description corresponding to the extrema of an objective function $J = f(H, \mu_{\tilde{O}}, \mu_{\tilde{B}})$. i.e.,

$$J = \sum_j h_j \mu_{\tilde{O}}(j) g(j, \xi_{\tilde{O}}) + \sum_j h_j \mu_{\tilde{B}}(j) g(j, \xi_{\tilde{B}}), \tag{17}$$

where $\xi_{\tilde{O}}$ is the set of parameters associated with the region $\tilde{O}$.

Fcm and pcm [14]: Fuzzy c-means-based thresholding formulations consider $g(\cdot)$ as a measure of scatter from the mean gray values of the regions, i.e.,

$$g(j, \xi_{\tilde{O}}) = (j - v_{\tilde{O}})^2.$$

The implementation is iterative and the convergence is analytically tractable and practically excellent. Both fcm and pcm-based formulations differ only in the membership assignment formulae.

Otsu [3]: Otsu also minimises a similar objective function in a hard setting, i.e.,

$$g(j, \xi_{\tilde{O}}) = (j - v_{\tilde{O}})^2 \quad \text{and} \quad \mu_{\tilde{O}}(j) \in \{0, 1\}.$$

The method basically searches the minima of the objective function by evaluation of the criteria function for all possible thresholds. These algorithms are optimum from a Bayesian sense when the object and background regions are equal in size and scatter.

Kittler and Illingworth [1]: Kittler and Illingworth assumed that the gray distributions corresponding to object and background regions are Gaussian in nature, i.e., the function $f(\cdot)$ in Eq. (6) is Gaussian. In this case,

$$g(j, \xi_{\tilde{O}}) = (p_j - N(j, \sigma_{\tilde{O}}, v_{\tilde{O}}))^2,$$

where $p_j = h_j / \sum_j h_j$ and $N(\cdot)$ is the Gaussian distribution.

Huang et al. *and Pal* [13,10]: These two methods basically minimise a fuzziness measure in the thresholded description. If fuzzy entropy is considered as such a measure

$$g(j, \xi_{\tilde{O}}) = \log(\mu_{\tilde{O}}(j)).$$

The mode of implementation is an extensive search for various parameters of object and background regions. The characteristic difference between these two methods lies in the philosophy of membership assignment.

Kapur et al. [2]: This is an important hard thresholding algorithm based on the Shannon's definition of entropy where minimisation of sum of entropies of object and background gray probabilities yield an optimal partition. Here,

$$g(j, \xi_{\tilde{O}}) = \log(p_j^{\tilde{O}}),$$

where $p_j^{\tilde{O}} = h_j / \sum_j \mu_{\tilde{O}}(j) h_j$ and $\mu_{\tilde{O}}(j) \in \{0, 1\}$. They also obtained the hard threshold by searching the minima of the objective function for all possible threshold values. It may be noted that the conventional thresholding schemes are only special cases of fuzzy thresholding when $\mu_{\tilde{O}}(j) \in \{0, 1\}$.

6.2. Performance characterisation

Segmentation provides means to compress the bulky raw image into a description based on the belongingness of the pixels to a set of regions. It is argued in the previous sections that a fuzzy thresholding scheme incorporate the details of the gray distribution of object and background regions. Thus it provides the details of gray distribution even after segmentation and thus become more useful for the high-level vision. In this case, the conventional performance characterisation based on classification accuracy may not be an appropriate choice. Here we propose a performance characterisation criteria

$$F = \sum_j h_j (\hat{\mu}_{\tilde{O}}(j) - \mu_{\tilde{O}}(j))^2 + \sum_j h_j (\hat{\mu}_{\tilde{B}}(j) - \mu_{\tilde{B}}(j))^2, \tag{18}$$

where $\hat{\mu}$ denotes the true membership and μ represents the membership assigned by the algorithm under consideration. In case of hard thresholding schemes $\mu(j) \in \{0, 1\}$. Since one may not know the exact memberships of gray values in natural scenes, we have considered a number of synthetic histograms as in our earlier work [14] and the fuzzy error measure is computed for a number of fuzzy and hard thresholding schemes. The results are shown in Table 1. Here the true membership is assumed to maximise at v_i and shows Gaussian characteristics with σ_i.

7. Discussions

Most of the thresholding algorithms are useful only when background and object regions are separable with gray values alone. Yet, all the thresholding algorithms are not found to perform equally well for all such scenes. The parameters ρ and γ play a crucial role in practice. It may be observed from formulations of algorithm 1 [14] that, when $\rho = \gamma = 1.0$ in Eq. (6), Bayesian optimal threshold coincides with T_v which is equidistant from the mean of the object and background regions. Since T_v is equidistant from v_1 and v_2, this algorithm is a favourable choice when regions are well balanced in size and scatter. In a more general case, this leads to the following lemma:

Lemma 1. *Thresholding schemes based on spherical clustering algorithms are not guaranteed to provide optimal (in Bayesian sense) thresholding when regions are not well balanced i.e.,* $\rho \neq 1$ *and/or* $\gamma \neq 1$.

For the histogram model Eq. (6), given $\sigma_1 = \sigma_2$, $T_v = (v_1 + v_2)/2$ is coincident with Bayesian optimal

Table 1
Performance of various thresholding schemes

Parameters			$F(\times 10^3)$							
σ_1	σ_2	ρ	Otsu	Kittler	Moment	Kapur	Huang	Murthy	FCM	PCM
15	15	1.00	10.67	10.67	10.67	10.67	8.061	8.061	6.137	0.644
15	15	0.50	10.68	10.73	11.18	11.18	8.063	8.063	6.139	0.646
15	15	0.33	10.68	10.79	11.25	11.25	8.064	8.064	6.141	0.648
15	5	1.00	10.68	10.80	20.47	20.47	8.889	8.889	6.771	4.053
15	10	1.00	10.68	10.73	11.16	11.16	8.454	8.454	6.319	1.792
15	10	0.50	10.69	10.69	10.80	10.80	8.586	8.586	6.379	2.178
15	10	0.33	10.69	10.69	10.88	10.88	8.654	8.654	6.409	2.376
15	5	0.33	10.69	10.72	10.70	10.70	9.308	9.308	7.094	5.771
15	5	2.00	10.68	10.90	20.63	20.63	8.611	8.611	6.567	2.914
15	5	3.00	10.68	10.93	17.73	17.73	8.473	8.473	6.463	2.346
15	10	3.00	10.68	10.86	12.16	12.16	8.262	8.262	6.232	1.212
15	10	2.00	10.67	10.84	12.32	12.32	8.321	8.321	6.260	1.410

threshold T_o, where background and object gray densities are equal, only if ρ is unity. Otherwise, if $\rho > 1.0$, T_o will be less than T_v and for $\rho < 1.0$, T_o will be greater than T_v. Indeed, it is assumed that the estimated v_1 and v_2 match with the true one, which generates the histogram.

The observations made here regarding the performance of thresholding schemes are valid for a general segmentation process, and applicability of such an algorithm for a wider class of images leads to the following assertion.

Performance of segmentation algorithms based on hyperspherical partitions depends on

(a) *overlap of regional density functions in the feature space,*
(b) *proportions in size of various regions, and*
(c) *scatter and type of distribution in the feature space.*

7.1. Multithresholding

The discussions carried out in the previous sections pertain to binary thresholding alone. The concepts and methods can be extended to a more general setting by considering the image to have c regions and each of them exhibiting distinct gray property. In such a case, the histogram becomes multimodal, and the segmentation process reduces to identification of valley points between the modes to partition the scene into distinct regions [6]. Fuzzy segmented description is achieved by minimising

$$\sum_{i=1}^{c}\sum_{j=1}^{L} h_j \mu_i^{\tau}(j) d^2(v_i, j). \tag{19}$$

with the help of fuzzy c-means or by finding the optimal possibilistic partition by minimising

$$\sum_{i=1}^{c}\sum_{j=1}^{L} h_j \mu_i^{\tau}(j) d^2(v_i, j) + \sum_{i=1}^{c} \eta_i \sum_{j=1}^{L} h_j (1-\mu_i(j))^{\tau}. \tag{20}$$

In both the cases, for $\tau > 1$, the segmented description is soft and reflects the details of the gray distribution. It may be noted that such a segmentation procedure does not guarantee the connectivity of pixels in a region. Fig. 5(a) depicts a scene consisting of distinct objects to be segmented. Fuzzy thresholding based on fuzzy clustering is employed for the purpose, and the resulting image is shown in Fig. 5(b) with different gray shades fpr distinct regions after hardening with scheme HARD-3. A possibilistic approach has also been tried for the thresholding. It has been found that both the algorithms are able to extract the modes of the histogram properly.

8. Summary

The classical thresholding schemes assign the pixel unequivocally to a region and do not distinguish among pixels in a region, even if their gray values are different in the original image. Consequently, the hard threshold selection schemes are associated with loss of structural details on thresholding. On the contrary, the identities of pixels are preserved in fuzzy partition space since the membership assigned to a pixel depends on the difference between its gray value and the mean gray value of the region to which it belongs. Fuzzy thresholding schemes can threshold noisy images too. Since thresholding schemes are, in general, sensitive to noise, fuzzy thresholding formulations also suffer in presence of severe

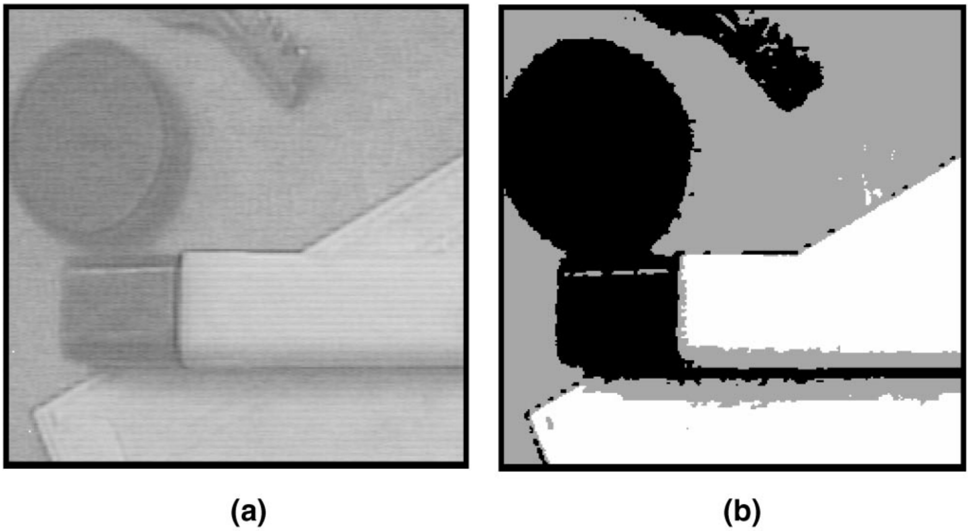

Fig. 5. Segmentation of a multi-modal scene.

noise. Performance of fuzzy algorithms in presence of noise requires further careful evaluation.

Fuzzy thresholding formulations based on fuzzy clustering have been extended to a possibilistic framework. The characteristic difference for assignment of membership and correspondence with conventional hard thresholding schemes have been investigated here. The possibility of unifying a number of hard and fuzzy thresholding schemes has been presented.

It may be observed that the hard thresholding schemes are only special cases of the fuzzy ones. Moreover, as far as the incorporation of the structural details of the gray distributions are concerned, fuzzy algorithms are superior to the conventional schemes.

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About the Author—C.V. JAWAHAR received B.Tech in 1991 from University of Calicut and M.Tech and Ph.D. from IIT Kharagpur in 1994 and 1997, respectively. Presently he is a scientist at the Centre for Artificial Intelligence and Robotics, Bangalore, India. His areas of interest include computer vision, fuzzy set theoretic approach to pattern recognition, texture analysis.

About the Author—P.K. BISWAS completed his B.Tech(Hons), M.Tech and Ph.D. in Electronics and Electrical Communication Engineering from Indian Institute of Technology, Kharagpur, in the years 1985, 1988 and 1991, respectively. From 1985 to 1987 he was with Bharat Electronics Ltd., Ghaziabad, India. Currently, he is an Assistant Professor in the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur. His areas of interest include pattern recognition, computer vision, parallel processing, distributed control and computer network.

About the Author—A.K. RAY received B.E in Electronics and Electrical Communication Engineering from B.E. College in 1975 and M.Tech and Ph.D. from IIT Kharagpur. From 1977 to 1980 he was associated with a DST sponsored research project on Automatic Postal Sorting and Biomedical Pattern Recognition'. He joined IIT Kharagpur in 1980 where he is currently an Associate Professor in the Department of Electronics and Electrical Communication Engineering. He has more than 50 research publications in National and International journals and conferences. His main research interest is on computer vision, pattern recognition and fuzzy system analysis. He has investigated a large number of sponsored and consultancy research projects in India on diverse aspects of static and dynamic scene analysis and automatic target tracking.